Roll No. ....

E–3905

B. C. A. (Part II) EXAMINATION, 2021

(Old Course)

Paper Second

DIFFERENTIATION AND INTEGRATION

(201)

Time : Three Hours] [*Maximum Marks : 50*

Note : Attempt any *two* parts from each question. All questions carry equal marks.

Unit—I

1. (a) If $y = \tan^{-1} x$, then prove that :

$$(1+x^2)\, y_{n+2} + 2(n+1)\, xy_{n+1} + n(n+1)\, y_n = 0 .$$

Hence find $(y_n)_0$.

(b) Verify Lagrange's mean value theorem for the function $f(x) = \sqrt{x^2 - 4}$ in the interval [2, 4].

(c) Find the first five terms in the expansion of log (1 + sin x) by Maclaurin's theorem.

Unit—II

2. (a) Show that the asymptotes of the cubic :

$$x^3 - 2y^3 + xy(2x - y) + y(x - y) + 1 = 0$$

cut the curve in three points which lie on a straight line $x - y + 1 = 0$.

(b) Determine the existence and nature of the double points on the curve :

$$(x-2)^2 = y(y-1)^2$$

(c) Trace the curve :

$$y = x(x^2 - 1)$$

Unit—III

3. (a) If :

$$u = \log(x^3 + y^3 + z^3 - 3xyz)$$

prove that :

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = -\frac{3}{(x+y+z)^2}.$$

(b) Evaluate the directional derivative of the function $\phi = x^2 - y^2 + 2z^2$ at the point P (1, 2, 3) in the direction of the line PQ, where Q has coordinates (5, 0, 4).

(c) Show that the functions :

$$u = x + 2y + z$$

$$v = x - 2y + 3z$$

$$w = 2xy - zx + 4yz - 2z^2$$

are not independent and find a relation between u, v, w.

Unit—IV

4. (a) Evaluate :

$$\int \frac{(x-a)(x-b)(x-c)}{(x-\alpha)(x-\beta)(x-\gamma)}\,dx$$

(b) Evaluate :

$$\int \cos^6 x\,dx$$

(c) Evaluate :

$$\int \operatorname{sech}^3 x\,dx$$

Unit—V

5. (a) Evaluate :

$$\iint_R e^{2x+3y}\,dx\,dy$$

where R is the region bounded by $x = 0$, $y = 0$ and $x + y = 1$.

(b) Change the order of integration in the following integral :

$$\int_{a}^{a\sqrt{2}} \int_{\cos^{-1}(a/y)}^{\pi/4} f(x, y)\, dx \; dy$$

(c) Find the perimeter of the cardioid :

$$r = a\,(1 - \cos\theta).$$