

Roll No.

E-3898

B. C. A. (Part II) EXAMINATION, 2021

(New Course)

Paper First

CALCULUS AND DIFFERENTIAL EQUATIONS

(BCA-201)

Time : Three Hours]

[Maximum Marks : 80

Note : Attempt any *two* parts from each Unit. All questions carry equal marks. Only simple calculator is allowed.

Unit—I

1. (a) Show that the following function $f(x)$ is continuous at $x = 0$ but $f'(0)$ does not exist :

$$f(x) = \begin{cases} \frac{xe^{1/x}}{1+e^{1/x}}, & \text{when } x \neq 0 \\ 0, & \text{when } x = 0 \end{cases}.$$

- (b) State and prove Mostest theorem.
(c) Test the continuity of the following function at $x = 0$:

$$f(x) = \begin{cases} \frac{\sin 2x}{x}, & \text{when } x \neq 0 \\ 1, & \text{when } x = 0 \end{cases}.$$

P. T. O.

Unit—II

2. (a) Find the maximum and the minimum values if any of the function :

$$f(x) = \sin 3x + 4;$$

$$x \in \left(-\frac{\pi}{2}, \frac{\pi}{2} \right)$$

- (b) If :

$$y = \sin(m \sin^{-1} x),$$

then prove that :

$$(1 - x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + m^2 y = 0.$$

- (c) Find $\frac{dy}{dx}$, where $y = x^{x^x}$.

Unit—III

3. (a) Evaluate :

$$\int \frac{dx}{3 - 2 \sin x}$$

- (b) Integrate :

$$\int x \tan^{-1} x dx$$

- (c) Integrate the following function w.r.t. x :

$$\int \frac{1}{x^2 + x + 1} dx$$

Unit—IV

4. (a) Show that :

$$\int_0^1 \frac{\log(1+x)}{1+x^2} dx = \frac{\pi}{8} \log 2$$

[3]

(b) Find the values of :

$$\int_0^1 \frac{dx}{\sqrt{1+x} + \sqrt{x}}$$

(c) Find the value of :

$$\int_0^\pi \frac{x \sin x}{1 + \cos^2 x} dx$$

Unit—V

5. (a) Discuss the general and particular solutions of a differential equation.

(b) Solve the differential equation $\frac{dy}{dx} = \frac{x}{y}$.

(c) Show that $v = \frac{A}{r} + B$ is a solution of differential

equation $\frac{d^2v}{dr^2} + \frac{2}{r} \frac{dv}{dr} = 0$.